When to stop searching in the future

something we hope to be better?

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Industrial applications

How to...

- Apply price discrimination to sell my products?
- Set different prices for consumers based on purchase history?
- Determine the personalized reserve price for Google ads?

Some facts

- The online data provider Lexis-Nexis sells to virtually every user at a different price.
- Orbitz online travel agency found that people who use Mac computers spent as much as 30% more on hotels.

Glance at the dynamic

A gambler is faced with a stream of numbers,
which are shown one by one.
She must select only one value
and only when it's first discovered.

Resumen

Mechanisms for auctions

1 item and n buyers.
Buyer i has valuations $v_i \in V$ and reveals message b_i.
A mechanism (q, p) consists in:
$q : B_1 \times \ldots \times B_n \to \Delta_0([n])$, allocation function.
$p : B_1 \times \ldots \times B_n \to \mathbb{R}^n$, payment functions.

Optimal mechanism

Assuming that $v_i \sim F_i$, the optimal mechanism (revenue maximizer) comes from solving the following optimization problem.

$$(P) \begin{cases} \max_q & \int_{V^n} q_i(v) \left(v_i - \frac{1-F_i(v_i)}{f_i(v_i)} \right) f(v) dv \\ s.t. & q(v) \in \Delta_0([n]) \\ & \mathbb{E}_{v_{-i}} \left[q_i(\cdot, v_{-i}) \right] \textit{ nondecreasing}. \end{cases}$$

Sequential offers

Offer the possible buyers,
sequentially in some order,
the item at a price τ_i.

How good can these mechanisms be?

$$\mathbb{E}(\text{Posted Price Mechanism}) \geq c\mathbb{E}(\text{Myerson Auction})$$

Another equivalent approach

Instead of maximize Revenue, take Social Welfare.
Instead of Myerson's Auction, take the maximum.
And the question is about

$$\mathbb{E}(ALG) \geq c\mathbb{E}(\max\{V_i\})$$

Rules of the game

1. Player one announces $F_1, \dots, F_n$.
2. Nature chooses, independently, a uniform random order σ.
3. Player two discovers sequentially $(V_{\sigma_1}, \sigma_1), \dots, (V_{\sigma_n}, \sigma_n)$, and takes one in an online fashion, where $V_{\sigma_i} \sim F_{\sigma_i}$ and are mutually independent.

The performance of player two is determined by

$$\frac{\mathbb{E}(ALG)}{\mathbb{E}(\max\{V_i\})} .$$

Optimal Strategy

Given by dynamic programming, or induction,

> "Take a value if it is better than
> what you expect to get in the future."

How good is this strategy?

IID. Case

When $F_1 = \ldots = F_n$,

$$\inf_F \frac{\mathbb{E}(ALG^*)}{\mathbb{E}(\max\{V_i\})} = c_n \searrow \approx 0.745 \,.$$

General Case

The best result so far is

$$\inf_n \inf_{F_1,\ldots,F_n} \frac{\mathbb{E}(ALG)}{\mathbb{E}(\max\{V_i\})} \geq 0.6697\,.$$

and, if ALG is nonadaptive,

$$\inf_n \inf_{F_1,\ldots,F_n} \frac{\mathbb{E}(ALG)}{\mathbb{E}(\max\{V_i\})} \leq 0.732\,.$$

Fixed Threshold Algorithm

For all continuous instances $F_1, \ldots, F_n$, there is a threshold τ such that

$$\frac{\mathbb{E}(ALG_\tau)}{\mathbb{E}(\max\{V_i\})} \geq 1 - \frac{1}{e} \approx 0.63$$

Proof.

Decreasing requirements over time

Can we do better by changing the threshold along the way?
How should we change it?
Choose $\tau_1, \dots, \tau_n$ such that

$$\mathbb{P}(\max\{V_i\} \leq \tau_i) = \alpha_i \,.$$

Under a good choice of,

$$\frac{\mathbb{E}(ALG_{\tau_1,\dots,\tau_n})}{\mathbb{E}(\max\{V_i\})} \geq 0.669 \,.$$

Proof?

References

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- U. Krengel, L. Sucheston, *On semiamarts, amarts and processes with finite value*, Adv. in Probability, 4 (1978), pp. 197-266.