When to stop searching in the future something we hope to be better?

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Industrial applications

How to...

- Apply price discrimination to sell my products?
- Set different prices for consumers based on purchase history?
- Determine the personalized reserve price for Google ads?

Some facts

- The online data provider Lexis-Nexis sells to virtually every user at a different price.
- Orbitz online travel agency found that people who use Mac computers spent as much as 30% more on hotels.

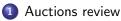
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Glance at the dynamic

A gambler is faced with a stream of numbers, which are shown one by one. She must select only one value and only when it's first discovered.

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Resumen



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- Prophet Inequality
- Dynamic
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 - How to extend the analysis

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Definition Myerson's auction Posted Price Mechanisms Prophet Inequality

Mechanisms for auctions

1 item and n buyers. Buyer i has valuations $v_i \in V$ and reveals message b_i . A mechanism (q, p) consists in: $q: B_1 \times \ldots \times B_n \to \Delta_0([n])$, allocation function. $p: B_1 \times \ldots \times B_n \to \mathbb{R}^n$, payment functions.

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Optimal mechanism

Assuming that $v_i \sim F_i$, the optimal mechanism (revenue maximizer) comes from solving the following optimization problem.

$$(P) \begin{cases} \max_{q} \quad \int_{V^n} q_i(v) \left(v_i - \frac{1 - F_i(v_i)}{f_i(v_i)}\right) f(v) dv \\ s.t. \quad q(v) \in \Delta_0([n]) \\ & \mathbb{E}_{v_{-i}} \left[q_i(\cdot, v_{-i})\right] \text{ nondecreasing.} \end{cases}$$

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Definition Myerson's auction **Posted Price Mechanisms** Prophet Inequality

Sequential offers

Offer the possible buyers, sequentially in some order, the item at a price τ_i .

How good can these mechanisms be?

 $\mathbb{E}(\text{Posted Price Mechanism}) \geq c\mathbb{E}(\text{Myerson Auction})$

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Definition Myerson's auction Posted Price Mechanisms Prophet Inequality

Another equivalent approach

Instead of maximize Revenue, take Social Welfare. Instead of Myerson's Auction, take the maximum. And the question is about

 $\mathbb{E}(ALG) \geq c\mathbb{E}(\max\{V_i\})$

Formal definition Results Simple proof How to extend the analysis

Rules of the game

- Player one announces F_1, \ldots, F_n .
- 2 Nature chooses, independently, a uniform random order σ .
- Player two discovers sequentially $(V_{\sigma_1}, \sigma_1), \ldots, (V_{\sigma_n}, \sigma_n)$, and takes one in an online fashion, where $V_{\sigma_i} \sim F_{\sigma_i}$ and are mutually independent.
- The performance of player two is determined by

 $\frac{\mathbb{E}(ALG)}{\mathbb{E}(\max\{V_i\})}\,.$

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Formal definition Results Simple proof How to extend the analysis

Optimal Strategy

Given by dynamic programming, or induction,

"Take a value if it is better than what you expect to get in the future."

How good is this strategy?

R. Saona Prophet Secretary problem

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Auctions review Dynamic Simple proof How to extend the analysis

When $F_1 = \ldots = F_n$,

$$\inf_{F} rac{\mathbb{E}(ALG^*)}{\mathbb{E}(\max\{V_i\})} = c_n \searrow pprox 0.745$$
 .

R. Saona Prophet Secretary problem

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Formal definition Results Simple proof How to extend the analysis

General Case

The best result so far is

$$\inf_{n} \inf_{F_1,\ldots,F_n} \frac{\mathbb{E}(ALG)}{\mathbb{E}(\max\{V_i\})} \geq 0.6697 \,.$$

and, if ALG is nonadaptive,

$$\inf_{n} \inf_{F_1,\ldots,F_n} \frac{\mathbb{E}(ALG)}{\mathbb{E}(\max\{V_i\})} \leq 0.732.$$

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Formal definition Results Simple proof How to extend the analysis

Fixed Threshold Algorithm

For all continuous instances F_1, \ldots, F_n , there is a threshold τ such that

$$rac{\mathbb{E}(ALG_{ au})}{\mathbb{E}(\max\{V_i\})} \geq 1 - rac{1}{e} pprox 0.63$$

Proof.

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Decreasing requirements over time

Can we do better by changing the threshold along the way? How should we change it? Choose τ_1, \ldots, τ_n such that

 $\mathbb{P}(\max\{V_i\} \leq \tau_i) = \alpha_i.$

Under a good choice of,

$$rac{\mathbb{E}(ALG_{ au_1,..., au_n})}{\mathbb{E}(\max\{V_i\})} \geq 0.669\,.$$

Proof?

References

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- U. Krengel, L. Sucheston, *On semiamarts, amarts and processes with finite value*, Adv. in Probability, 4 (1978), pp. 197-266.

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